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Time Delay Compensation and Stability Analysis of Networked Predictive Control Systems Based on Hammerstein Model

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Abstract— A novel approach is proposed for a networked control system with random delays containing a nonlinear process based on a Hammerstein model. The method uses a Time Delay Two Step Generalized Predictive Control(TDTSGPC), which consists of two parts, one is to deal with the input nonlinearity of the Hammerstein model and the other is to compensate the network induced delays in the networked control system. Theoretical results using the Popov theorem are presented for the closed-loop stability of the system in the case of a constant delay. Simulation examples illustrating the validity of the approach are presented.

I. INTRODUCTION

A control loop is called a "Networked Control System" (NCS) when it is closed via a serial communication network[1].This configuration brings to the system lower cost, flexibility, the ability of remote control, etc., but at the same time, the time delay introduced by the network (so called "network-induced delay") greatly degrades the performance of the system, even makes the system unstable under certain conditions. Such an implementation presents a new challenge to conventional control theory.

A large number of papers have addressed NCSs to date, but unfortunately, only some basic cases have been considered or some unrealistic assumptions have been made [2], [3], [4], for example:

1) Basic system structures, mainly linear systems;

2) Simple assumptions of the characteristics of network-induced delays, mostly constant.

In this paper, the input nonlinearity (represented by Hammerstein model) and a random network-induced delay are considered for NCSs. A Time Delay Two-Step Generalized Predictive Control (TDTSGPC) approach is proposed, which includes two parts, one is a Two-Step Generalized Predictive Controller (TSGPC) [5], [6] to deal with the input nonlinearity and the other is a time delay compensator to deal with the network-induced delay. The paper considers the stability analysis of the corresponding closed-loop system. Simulations are also done to illustrate the validity of the approach.

The remainder of this paper is organized as follows. The design of TDTSGPC based on a Hammerstein model is presented in Section 2. Then the theoretical results for the system stability and the simulation results of TDTSGPC are presented in Section 3 and Section 4, respectively. The paper gives the conclusions in Section 5.

II. DESIGN OF TDTSGPC BASED ON HAMMERSTEIN MODEL

The Hammerstein model, which consists of the cascade connection of the static nonlinearity followed by a dynamic LTI system, is a common model in control theory. Here the static nonlinearity of the Hammerstein model is represented by $v(k) = f(u(k))$ where $f(\cdot)$ is a nonlinear function with $f(0) = 0$ and the linear part is represented by CARIMA model:

$$ay(k) = bv(k-1) + \xi(k)/\Delta \qquad (1)$$

where $\xi(k)$ is Gaussian white noise with zero mean value, $\Delta = 1 - z^{-1}$, $a = 1 + a_1 z^{-1} + ... + a_n z^{-n}$, $b = b_0 + b_1 z^{-1} + ... + b_m z^{-m}$ with $a_n \neq 0, b_m \neq 0$.

In the following section, the two parts of TDTSGPC, the design of TSGPC and the time delay compensator, will be presented respectively.

A. The design of TSGPC

The key idea of TSGPC is to design the intermediate control sequence $v(k)$ of the linear part of Hammerstein model (1) with linear GPC method (LGPC) first, and then obtain the real control sequence $u(k)$ from the relationship $v(k) = f(u(k)), k = 1, 2, ...N_2$ where N_2 is the control horizon [6].

1) The design of LGPC: Without consideration of the input nonlinearity of the Hammerstein model, solve the LGPC problem for (1) and objective function:

$$minJ(k) = ||Y(k) - \varpi||_Q^2 + ||\Delta V(k)||_R^2 \qquad (2)$$

where $Y(k) = \begin{bmatrix} y(k+1) & y(k+2) & ... & y(k+N_1) \end{bmatrix}^T$, $y(k + i), i = 1, 2, ...N_1$ are the outputs, $\varpi = \begin{bmatrix} \omega & \omega & ... & \omega \end{bmatrix}^T$, ω is the set-point, $Q = diag\begin{pmatrix} q_1 & q_2 & ... & q_{N_1} \end{pmatrix}$, $R = diag\begin{pmatrix} r_1 & r_2 & ... & r_{N_2} \end{pmatrix}$, $q_i, i = 1, 2, ..., N_1, r_i, i = 1, 2, ..., N_2$ are the weight coefficients, $\Delta V(k) =$

$\begin{bmatrix} \Delta v(k) & \Delta v(k+1) & ... & \Delta v(k+N_2-1) \end{bmatrix}^T$ and $||\psi||^2_\Theta$ means $\psi^T \Theta \psi$.

Introduce the following Diophantine equations

$$1 = E_j a \Delta + z^{-j-\tau^{sc}} F_j, E_j b = z^{-j} E_j^0 + G_j, j = 1, 2, ..., N_1 \tag{3}$$

where τ^{sc} is the time delay of feedback channel and $E_j = 1 + e_{j,1}z^{-1} + ... + e_{j,j+\tau^{sc}-1}z^{-(j+\tau^{sc}-1)}$, $F_j = f_{j,0} + f_{j,1}z^{-1} + ... + e_{j,n}z^{-n}$, $E_j^0 = e_{j,0}^0 + e_{j,1}^0 z^{-1} + ... + e_{j,m+\tau^{sc}-1}^0 z^{-(m+\tau^{sc}-1)}$, $G_j = g_{j,0} + g_{j,1}z^{-1} + ... + g_{j,j-1}z^{-(j-1)}$, and define $E = \begin{bmatrix} E_1^0 & E_2^0 & ... & E_{N_1}^0 \end{bmatrix}^T$, $F = \begin{bmatrix} F_1 & F_2 & ... & F_{N_1} \end{bmatrix}^T$, $G \in R^{N_1 \times N_2}(z^{-1})$ with $G(j,j) = G_j$ for $j = 1, ..., N_2$ and all the other entries are 0, $D = (G^T Q G + R)^{-1}G^T Q$, $Y_0(k|k - \tau^{sc}) = E\Delta v(k-1) + Fy(k - \tau^{sc})$, $M = \begin{bmatrix} 1 & 1 & ... & 1 \end{bmatrix}^T_{N_2 \times 1}$, $V(k|k - \tau^{sc}) = \begin{bmatrix} v((k|k-\tau^{sc}) & v(k+1|k-\tau^{sc}) & ... & v(k+N_2|k-\tau^{sc}) \end{bmatrix}^T$,

$$C = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{N_2 \times N_2}$$, then the predictive control

sequence under objective function (2) at time k based on the outputs before $k - \tau^{sc}$ and the previous control sequence is

$$V(k|k - \tau^{sc}) = Mv(k-1) + CD(\varpi - Y_0(k|k-\tau^{sc})) \tag{4}$$

For more details of the calculation of predictive control sequences, the reader is referred to [7] and the references therein.

2) The input nonlinearity: Assume $f(\cdot)$ is invertible then a nonlinear function $f^{-1}(\cdot)$ exists such that

$$u(k) = f^{-1}(v(k)) \tag{5}$$

Thus, at every time instant k, the intermediate input $v(k), k = 1, 2, ..., N_2$ are obtained from (4), and then the real input $u(k), k = 1, 2, ..., N_2$ can be calculated from (5) using a numerical method thus enabling the control law to be defined.

If the real input $u(k)$ can be calculated perfectly accurately by (5), then TSGPC is equivalent to LGPC and the system is stable if and only if the linear system (1) is stable. However, in practice, it is usually impossible to calculate $u(k)$ that accurately, and therefore difficulties are introduced in the stability analysis of the closed-loop system. Here, the practical inverse of $f(\cdot)$ is denoted by $\hat{f}^{-1}(\cdot)$ and usually $f \cdot \hat{f}^{-1} \neq 1$. For more information of the calculation of $f(\cdot)$, one can refer to [8] and the references therein.

B. The design of time delay compensator

The network introduces to the NCSs delays which greatly degrade the performance of the system, even making the system unstable under certain conditions, while at the same time, the network also brings an advantage to the system in that a sequence of signals can be packed and transmitted simultaneously[9], [10]. Our time delay compensator takes advantage of this property of NCSs.

The following assumptions are made in the time delay compensation scheme:

1) For the sake of the calculation of the predictive control sequence, the time delay of the feedback channel needs to be known to the controller, which can be easily done by issuing a time stamp on each data package from the sensor side to the controller side;

2) The Round Trip Time (RTT, noted by τ, the total time delay of feedback channel and forward channel, i.e. $\tau = \tau^{sc} + \tau^{ca}$) is known to the actuator, which can also be done by using the time stamps;

3) The predictive control sequences are packed and transmitted to the actuator simultaneously;

4) The forward time delay is less than the control horizon N_2.

The time delay compensator works as follows: at every time instance k, the predictive controller calculates a sequence of future control signals based on the outputs before $k - \tau_k^{sc}$ (the time delay of feedback channel at time k) and the previous control sequence. The future control signals are then transmitted to the actuator side with a time stamp k all in one package. When a package of a control sequence arrives at the actuator side (different packages may experience different time delays), it is compared with the one already in the cache of the actuator according to the time stamp and only the new one is reserved. The actuator then chooses the control action $u(k + \tau_k^{ca}|k - \tau_k^{sc})$ if the time stamp of the control sequence in its cache is k and the forward time delay is τ_k^{ca}.

The TDTSGPC approach can be represented by Fig.1, where $d_{\tau^{ca}}^T \in R^{N_2 \times 1}$ is a column vector in which only the τ^{ca}th entry is 1 while other entries are all 0, $d^T = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}_{N_2 \times 1}$, and $\hat{\bar{f}}^{-1}(\cdot) = \begin{bmatrix} \hat{f}^{-1}(\cdot) & \hat{f}^{-1}(\cdot) & \cdots & \hat{f}^{-1}(\cdot) \end{bmatrix}^T_{N_2 \times 1}$.

III. STABILITY ANALYSIS

In this section, the Popov theorem is applied to prove the stability of the TDTSGPC approach.

Lemma 1. (Popov Theorem, see [11]) Suppose that $H(z)$ in Fig.2 is stable and $0 \leq \Phi(\theta) \leq K\theta$, then the closed-loop system is stable if $1/K + Re(H(z) > 0, \forall |z| = 1$.

In the case of constant delays, apply Lemma 1 to TDTS-GPC and denote the characteristic polynomial of the transfer function of the feedforward channel in Fig.3 by $\delta(H)$, we then obtain the following theorem.

Theorem 1. Suppose that the linear part of the Hammerstein model is accurate and the roots of $\delta(H) = 0$ are located in the unit circle, then the closed-loop system of TDTSGPC is stable if there exist a positive constant K such that

1) the input nonlinearity of the plant satisfies

$$0 \leq v \leq K\bar{v},$$

2) the time delay τ satisfies

$$\frac{1}{K} + Re\{\frac{z^{-\tau-1}d_{\tau^{ca}}^T D(I + Ed^T D)^{-1}Fb}{\Delta a}\} > 0, \forall |z| = 1$$
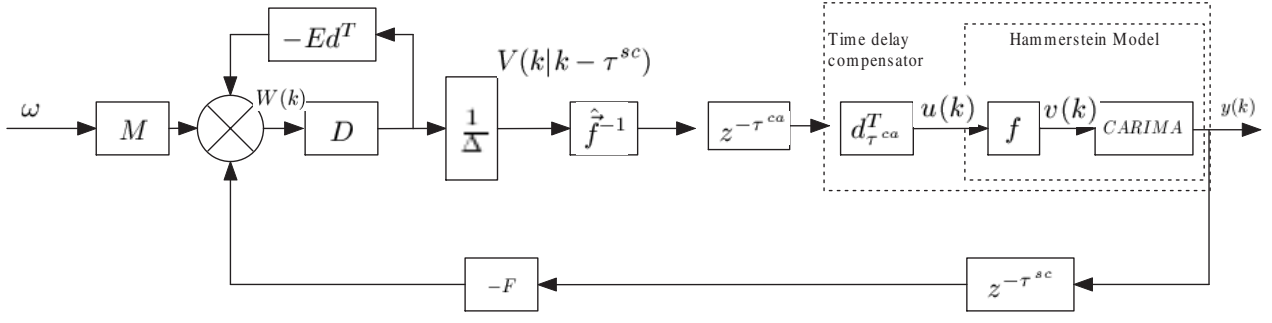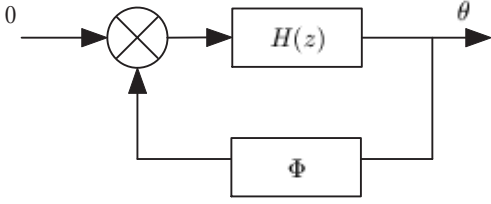
Fig. 1. The block diagram of TDTSGPC
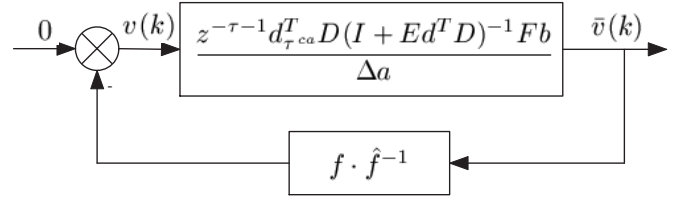

Fig. 2. Popov Theorem

Fig. 3. The simplified block diagram of TDTSGPC

Proof. Similar to the proof of Theorem 1 in [11], w.l.o.g, assume $\omega = 0$. The block diagram of TDTSGPC shown in Fig.1 can be fully described by the following equations:

$$
\begin{cases}
ay(k) = bv(k-1) & (6) \\
v(k) = f(u(k)) & (7) \\
u(k) = d_{\tau^{ca}}^T \hat{\tilde{f}}^{-1}(\frac{D}{\Delta}W(k-\tau^{ca})) & (8) \\
W(k) = M\omega - Fy(k-\tau^{sc}) - Ed^T D W(k) & (9)
\end{cases}
$$

It is easy to see $W(k) = (I+Ed^T D)^{-1}(M\omega - Fy(k-\tau^{sc})$ from (9). Notice here $\omega = 0$ and for any column vector P with comparable dimensions, $f(d_{\tau^{ca}}^T \hat{\tilde{f}}^{-1}(P)) = f \cdot \hat{f}^{-1}(d_{\tau^{ca}}^T P)$ recalling the definition of $\hat{\tilde{f}}^{-1}(\cdot)$, then from equations (6)-(8) and the expression of $W(k)$ above, we obtain

$$
\begin{aligned}
v(k) =& f(u(k)) \\
=& f(d_{\tau^{ca}}^T \hat{\tilde{f}}^{-1}(\frac{D}{\Delta}W(k-\tau^{ca})) \\
=& f \cdot \hat{f}^{-1}(-\frac{d_{\tau^{ca}}^T D}{\Delta}W(k-\tau^{ca})) \\
=& f \cdot \hat{f}^{-1}(-\frac{d_{\tau^{ca}}^T D}{\Delta}(I+Ed^T D)^{-1}Fy(k-\tau)) \\
=& f \cdot \hat{f}^{-1}(-\frac{z^{-\tau-1}d_{\tau^{ca}}^T D(I+Ed^T D)^{-1}Fbv(k)}{\Delta a}) \quad (10) \\
=& f \cdot \hat{f}^{-1}(\bar{v}(k)) \quad (11)
\end{aligned}
$$

where $\bar{v}(k)$ is the theoretical input value to the CARIMA model and the real input $v(k) = f \cdot \hat{f}^{-1}(\bar{v}(k))$ from (11). This is equivalent to the block diagram shown in Fig.3. Thus the theorem can be easily obtained by applying Lemma 1 to Fig.3.

IV. SIMULATIONS

Here we give an example to illustrate the TDTSGPC approach. The linear part of the system adopted is $y(k) -$

$0.5y(k-1) = v(k)$, and the input nonlinearity of Hammerstein model is chosen as $v = f(u) = u^2$ and the practical inverse of $f(\cdot)$ is $\hat{f}^{-1} = \sqrt{v} \times \epsilon$, where ϵ is a random number with a uniform distribution in $[0\ 1]$. This is introduced to represent the uncertainty in a practical implementation. From condition (1) of Theorem 1 we see the parameter K is 1 also the predictive horizon and control horizon are chosen as $N_1 = N_2 = 8$.

It can be shown that the system is stable only for the first two cases according to Theorem 1 since too large a time delay will make the system not satisfy condition (2) in Theorem 1. The simulation results of three cases: i) $\tau^{sc} = \tau^{ca} = 0$; ii) $\tau^{sc} = 1, \tau^{ca} = 2$; and iii) $\tau^{sc} = 4, \tau^{ca} = 2$ are shown in Fig.4-6 and illustrate the validity of the theoretical analysis.
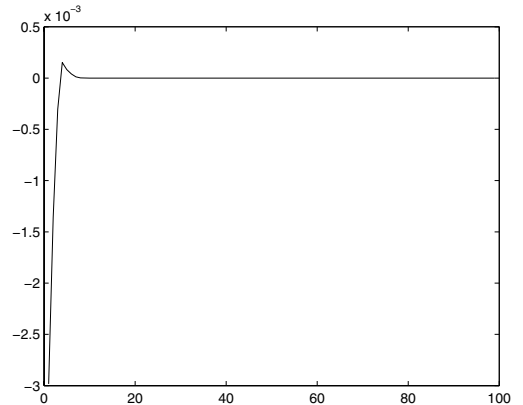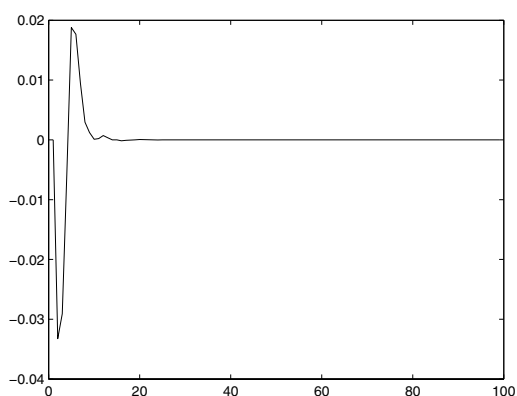

Fig. 4. $\tau^{sc} = 0, \tau^{ca} = 0$
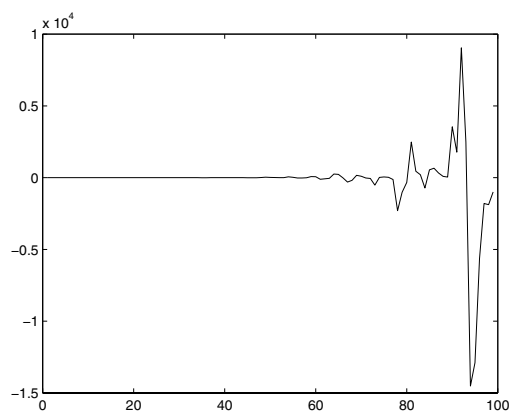
Fig. 5. $\tau^{sc} = 1, \tau^{ca} = 2$

Fig. 6. $\tau^{sc} = 4, \tau^{ca} = 2$

V. CONCLUSIONS

In this paper , the Two Step Generalized Predictive Control approach, which is often used in the controller design for the Hammerstein model system, is integrated with a time delay compensator to deal with the networked control systems based on Hammerstein model with random network-induced delays. This novel approach takes advantage of the characteristic of networks that a sequence of information can be packed to transmit simultaneously so that predictive control method can be really applied to NCSs. A theoretical result is presented for the stability of the system in the case of constant time delay. Simulation work has also been done to illustrate the validity of the approach. Further research is still needed to analyze the stability conditions under random time delays which is not addressed in this paper.

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